Games, graphs, and machines

September 17, 2024

Stars and cats = con CAT enation

Alphabet $\Sigma = \{0, 1\}$.

Languages $L = \{0\}$ and $M = \{1, 11, 111, 1111, \cdots\}$.

NOT the same
vs $\{1\}^*$
↑ includes ε

1. $LM =$ $L \circ M = \{01, 011, 0111, \cdots\}$
2. $ML =$ $\{10, 110, 1110, \cdots\}$
3. $L^* =$ $\{\varepsilon, 0, 00, 000, \cdots\}$
4. $M^* =$ $\{\varepsilon, 1, 11, 111, \cdots\}$
5. $L^*M =$ $\{1 \leftarrow \varepsilon \cdot 1$ (L^*, M)

0 or more 0s followed by 1 or more 1s

01, 11
001, 011, 111
0001, 0011, 0111, 1111,
$\cdots\}$

man grep

```
REGULAR EXPRESSIONS
       A regular expression is a pattern that describes a set of strings.
       Regular expressions are constructed analogously to arithmetic
       expressions, by using various operators to combine smaller expressions...

Character Classes and Bracket Expressions
       A bracket expression is a list of characters enclosed by [  and  ]...

Anchoring
       The  caret ^ and the dollar sign $ are meta-characters...

The Backslash Character and Special Expressions
       The symbols \< and  \>  respectively  match  the  empty  string...

Repetition
       A  regular  expression  may  be  followed  by  one of several repetition
       operators:
       ?      The preceding item is optional and matched at most once.
       *      The preceding item will be matched zero or more times.
       +      The preceding item will be matched one or more times.
       {n}    The preceding item is matched exactly n times.
       {n,}   The preceding item is matched n or more times.
       {,m}   The preceding item is matched at most m times.   This  is  a  GNU
              extension.
       {n,m}  The preceding item is matched at least n times, but not more than
              m times.

Concatenation
       Two  regular  expressions  may  be  concatenated;  ...

Alternation
       Two  regular  expressions  may  be  joined  by the infix operator |;...
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Our regexps

- $\varnothing$
  ϵ
  0
  1
- Concatenation ab
  alternation $a|b$
  star a^*.

$(\varepsilon|0)^*\ (0|1)$

* precedes

concat precedes

|

01^*

$01^*\ |\ 10^*$

$(01)^*$

$(0\,(1^*))\ |\ (1\,(0^*))$

Regular expressions

Explicitly write the language described by the regexp.

1. 01^* $= 0 \cdot 1^* = \{0\} \cdot \{\varepsilon, 1, 11, 111, \dots\} = \{0, 01, 011, \dots\}$
2. $(0|1)^*$ $= \{0,1\}^* =$ All Strings
3. $(01)^*$ $= \{\varepsilon, 01, 0101, \dots\}$
4. 00^*10^*0

$0 \cdot 0^* \cdot 1 \cdot 0^* \cdot 0$

$0 \cdot \{\varepsilon, 0, 00, \dots\} \cdot 1 \cdot \{\varepsilon, 0, 00, 000, \dots\} \cdot 0$

$0\,\underbrace{0\cdots0}_{0 \text{ or more}}\,1 \cdot \underbrace{0\cdots0}_{0 \text{ or more}}\,0 \qquad \underbrace{0\,0}_{1 \text{ or more}}\,1\,\underbrace{0\cdots0}_{1 \text{ or more}}$

Building regexps

Find regular expressions that describe the following languages.

1. $\emptyset$ → ϕ
2. $\{\epsilon\}$ → ε
3. $\{0, 00, 000, \cdots\} = L(00^*) = L(00^*) = L(0|000^*)$
4. $\{w \mid w \text{ starts with 0 and ends with 1}\}$

11

$0\,0^*\,1^*\,1$

$0\,1^*$

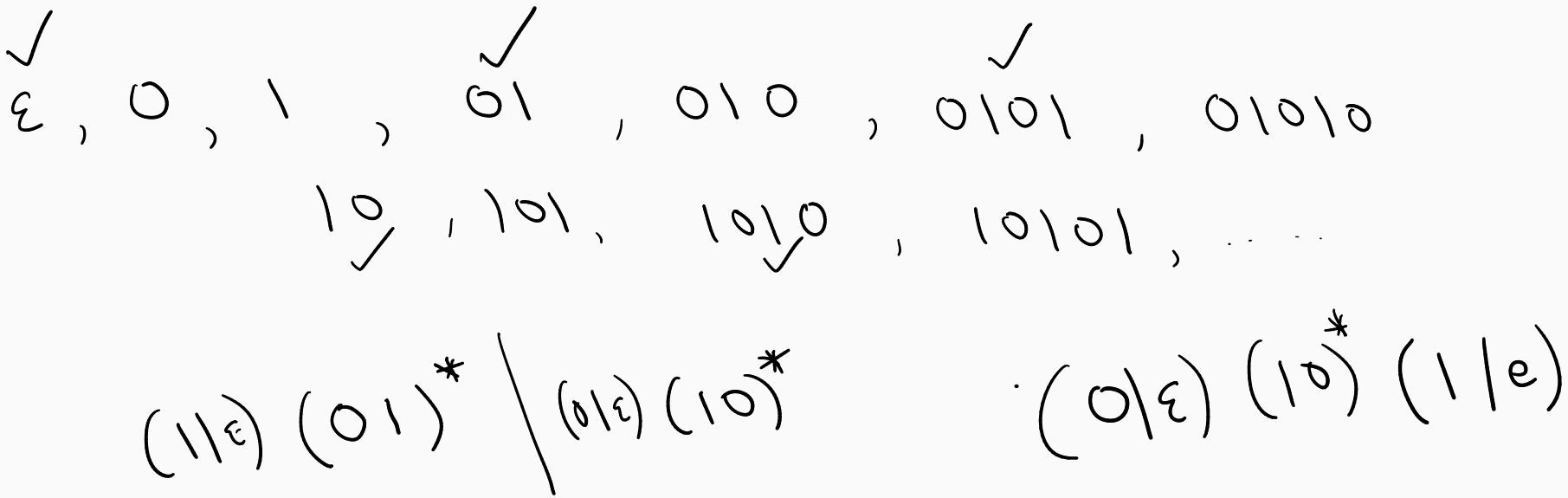
$0\,(0|1)^*\,1$

0101

Building trickier regexps

Find regular expressions that describe the following languages.

1. $\{w \mid 0 \text{ and } 1 \text{ alternate in } w\}$
2. $\{w \mid \text{every } 0 \text{ in } w \text{ has } 1 \text{ on its left and on its right}\}$.
3. $\{w \mid w \text{ has an even number of } 0\text{s}\}$



Even trickier languages

Can you find regexps that describe the following languages?

1. $\{w \mid w$ has as many 0s as 1s$\}$.
2. $\{w \mid w$ is a palindrome$\}$.